

جمهورية مصر العربية



وزارة التربية والتعليم
والتعليم الفني

نموذج إجابة

امتحان شهادة إتمام الدراسة الثانوية العامة

للعام الدراسي ٢٠١٧/٢٠١٦ - الدور الأول

المادة : التفاضل والتكامل (باللغة الانجليزية)

نموذج



لكل مجموعة مقدار مراجع

الدرجة	الأسئلة من إلى
٧	١ ← ٤
٥	٥ ← ٦
٧	٧ ← ١١
٥	١٢ ← ١٤
٦	١٥ ← ١٨
٣.	المجموع

1-

$$(a) x + \frac{1}{2} \sin 2x + c$$



2-

Let , $OA = x$ and $OB = y$

$$\therefore AD = x - 3$$

From the similarity of the two triangles DAC and OAB we found that

$$\frac{x-3}{x} = \frac{2}{y}$$

$$\therefore y = \frac{2x}{x-3}$$

$$\text{Area of } \Delta OAB = \frac{1}{2} xy$$

$$A = \frac{1}{2} \times x \times \frac{2x}{x-3} = \frac{x^2}{x-3}$$

$$A' = \frac{2x(x-3)-x^2}{(x-3)^2}$$

$$\therefore \text{at the least area } A' = 0$$

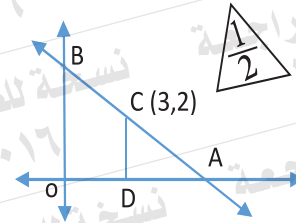
$$\therefore 2x^2 - 6x - x^2 = 0$$

$$x^2 - 6x = 0$$

$$\therefore x = 0 \quad \text{Oder} \therefore x = 6$$

$$\therefore \text{the area is minimum at } x = 6$$

$$\therefore \text{the smallest area} = \frac{6^2}{6-3} = 12 \text{ area unit}$$



3-

(a) 4



4-

The Points of intersection

$$x^2 = 5x$$

$$x^2 - 5x = 0$$

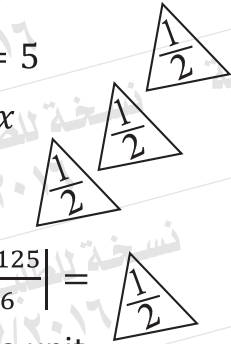
$$\therefore x = 0 \text{ or } x = 5$$

$$A = \int_0^5 |x^2 - 5x| dx$$

$$= \left| \frac{x^3}{3} - \frac{5x^2}{2} \right|_0^5$$

$$= \left| \frac{125}{3} - \frac{125}{2} \right| = \left| -\frac{125}{6} \right| = \frac{125}{6}$$

$$\therefore \text{Area} = \frac{125}{6} \text{ area unit}$$



5-

The Points of intersection

$$x^2 = 3x$$

$$x^2 - 3x = 0$$

$$x = 0, x = 3$$

$$v = \pi \int_0^3 |x^4 - 9x^2| dx$$

$$= \pi \left| \frac{x^5}{5} - 3x^3 \right|_0^3$$

$$= \pi \times \left| \frac{3^5}{5} - 3 \times 3^3 \right|$$

$$\frac{162}{5} \pi \text{ Volume Unit}$$

6-

$$(a) \int \frac{x+1-1}{x+1} dx$$

$$= \int \left(1 - \frac{1}{x+1} \right) dx$$

$$= x - \ln|x+1| + c$$

$$(b) \int x^2 \ln x dx$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 dx$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + c$$

7-

(d) $f(-2)$



8-

(c) $2x + c$



9-

(a) $\ln |\sin x| + c$



10-

$\therefore y = 3e^x$

$\therefore \dot{y} = 3e^x$ at $x = -1$, $y = 3e^{-1} \therefore \text{slope of tangent} = 3e^{-1} = \frac{3}{e}$

$\therefore \text{the equation of the normal is : } y - y_1 = \frac{-1}{\text{slope}} (x - x_1)$

$\therefore y - 3e^{-1} = -\frac{e}{3} (x + 1)$

$\therefore y = \frac{3}{e} - \frac{ex}{3} - \frac{e}{3}$

11-

(a) $\frac{-\pi}{4}$



12-

$$(c) \frac{-1}{6}$$



13-

$$\because x \times y = \frac{z+1}{z-1} \times \frac{z-1}{z+1} = 1$$



$$\therefore y = \frac{1}{x}$$

$$y = x^{-1}$$



$$y' = -x^{-2}$$

$$y'' = 2x^{-3} \Rightarrow (1)$$



$$\text{At } z = \text{zero} \therefore x = -1 \Rightarrow (2)$$

$$\text{by Substitution in (1)} \frac{d^2y}{dx^2} = 2 \times (-1)^{-3} = -2$$



Another
Solution

$$\frac{dx}{dz} = \frac{z-1-z-1}{(z-1)^2} = \frac{-2}{(z-1)^2}$$

$$\frac{dy}{dz} = \frac{z+1-z+1}{(z+1)^2} = \frac{2}{(z+1)^2}$$



$$\therefore \frac{dy}{dx} = \frac{-(z-1)^2}{(z+1)^2}$$



$$\frac{d^2y}{dx^2} = \frac{-2(z-1)(z+1)^2 - 2(z+1)(-(z-1)^2)}{(z+1)^4} \times \frac{(z-1)^2}{-2}$$



$$\text{At } z=0 \quad \frac{d^2y}{dx^2} = \frac{(-2)(-1)(1)^2 - 2(1)(1)(-1)}{(1)^4} \times \frac{1}{-2} = -2$$



14-

$$\therefore A = \pi r^2$$



$$\therefore \frac{dA}{dt} = 2\pi r \times \frac{dr}{dt}$$



$$\text{After 5 seconds } r = 4 \times 5 = 20 \text{ cm}$$



$$\therefore \frac{dA}{dt} = 2\pi \times 20 \times 4$$

$$= 160\pi \text{ cm}^2/\text{sec}$$



15-

(a) 4



16-

(b) $-\frac{1}{4}$



17-

(d) $\sqrt{2}$



18-

(a)

The domain of the function is \mathbb{R} .

$$f(x) = (2 - x)e^x$$

$$f'(x) = -e^x + (2 - x)e^x$$

$f'(x) = 0$ at the critical points

$$\therefore -e^x + (2 - x)e^x = 0$$

$$\therefore -1 + 2 - x = 0 \quad \therefore x = 1$$

$$f''(x) = -e^x - e^x + (2 - x)e^x \\ = -2e^x + (2 - x)e^x$$

$$f''(1) = -2e + e = -e = \text{negative}$$

\therefore There is a maximum value at $x = 1$ equals e

(b)

$$\therefore f(x) = 3x^4 - 4x^3$$

$$\therefore f'(x) = 12x^3 - 12x^2$$

$$\therefore f'(x) = 0$$

$$\therefore 12x^2(x - 1) = 0$$

$$\therefore x = 0 \in [-1, 2]$$

$$\text{or } x = 1 \in [-1, 2]$$

$$f(0) = 3 \times 0^4 - 4 \times 0^3 = 0$$

$$f(1) = 3 \times 1^4 - 4 \times 1^3 = -1$$

$$f(-1) = 3(-1)^4 - 4(-1)^3 = 7$$

$$f(2) = 3(2)^4 - 4(2)^3 = 16$$

The minimum value is -1 , the maximum value is 16

(انتهت الإجابة وتراعى الحلول الأخرى)